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Transonic Airfoil Design

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A method recently developed by Northrop consists of a computer program which will determine an airfoil shape from predetermined supercritical velocity distributions having extensive regions of supersonic flow. The velocity is given vs the distance around the airfoil. This allows a designer to design to a given lift by specifying the required circulation. Also, boundary-layer problems can be avoided by restricting adverse velocity gradients. Starting with a given compressible pressure or velocity distribution with mixed subsonic and supersonic regions, an airfoil shape can be determined. This is done by making a transformation that causes the streamline and potential line network to give an equivalent incompressible flow. This incompressible problem is then solved by complex function theory, and the solution is transformed back to the compressible plane. A computer program using this method has been applied to several shapes with known solutions. Agreement between calculated shapes and actual shapes was excellent. A 4.6% airfoil was designed from a prescribed velocity distribution and tested in the wind tunnel at transonic speeds. Good agreement was obtained between theoretical and experimental results. Transonic airfoils can be designed by this method.

Nomenclature

a^* = speed of sound at Mach one
C_L = lift coefficient
C_M = pitching moment coefficient
C_P = pressure coefficient
Im = imaginary part
M = Mach number
n = distance normal to streamline
Re = real part
s = distance along streamline
V = local velocity
V_∞ = freestream velocity
Z_c = complex coordinate in compressible plane
Z_i = complex coordinate in incompressible plane
α = local flow angle in airfoil plane
ϵ_T = trailing-edge angle
ζ = complex coordinate in circle plane
θ = angle in circle plane
ρ = density
ρ_0 = stagnation density
ϕ = velocity potential
ψ = stream function

Subscripts

c = compressible
i = incompressible

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Introduction

IF aircraft flight speed approaches the speed of sound, the air flowing around the aircraft will be mixed supersonic and subsonic flow. The equations for calculating the flow and its resultant effects on the forces on a body are radically different for supersonic and subsonic speeds. When a flow problem consists of a mixture of both types of flow, the resulting problem becomes very difficult. As a result, scientists and engineers concerned with the development of modern aircraft rely heavily on empirical techniques and wind-tunnel test results.

The state-of-the-art today is rapidly progressing. Several years ago Pearcey[1] at the National Physics Laboratory in England noticed that some wings being tested in the wind tunnel exhibited at certain conditions the property of inducing supersonic flow over its surface and decelerating the air back to subsonic speeds with apparently no accompanying shock waves. Capitalizing on his observation, Pearcey was able to devise empirical rules which allowed the apparently shock-free flow to be obtained at will. Later, Whitcomb[2] at NASA, using empirical techniques, designed an unusual wing with very promising characteristics at high subsonic speeds. Nieuwland[3] at the National Aerospace Laboratory in Amsterdam developed a theoretical method for the exact calculations of transonic flow about a certain class of wing shapes and obtained experimental verification of his results. Other workers, such as Yoshihara[4] at General Dynamics, have developed elegant, although laborious, methods for the solution to the transonic flow problem that include the effects of viscosity. Whether or not shocks are present on the wings of recent airfoil designs is still a debatable issue; however, if the shocks are weak and have little effect, aircraft designers are satisfied.

The design of transonic wings for airplanes is still mostly an art where likely shapes are selected, wind-tunnel tested, and modified until what is considered an optimum compromise is achieved. A promising analytical approach has been developed by Cahn, Wasson, and Wooler. This approach has been experimentally verified by Andrew and Garcia.

Outline of Theory

The basic problem to be solved is the determination of a ϕ vs ψ orthogonal network, which has the property that[5]

$$\partial\phi/\partial s = (1/\rho)(\partial\psi/\partial\eta)$$

This network is made up of rectangles which have a length to height ratio equal to the local density (see Fig. 1). If this network were distorted so that the rectangles become squares (see Fig. 2), the problem would become incompressible and would be solved by classic complex function techniques.

It is assumed that the compressible velocity V_c vs airfoil surface distance s_c is given. From the given V_c vs s_c, the compressible velocity potential function ϕ_c can be calculated on the boundary of the airfoil. An equivalent incompressible airfoil is presumed to exist with the same velocity potential but having a different shape. The change in airfoil shape is in the form of a surface stretching, the local flow angle α remaining the same in the two planes. Now

$$\phi_c = \int V_c ds_c$$

and

$$\phi_i = \int V_i ds_i$$

so that

$$\phi_c = \phi_i$$

implies

$$ds_i/ds_c = V_c/V_i$$

To get the incompressible ϕ_i vs s_i distribution, it is necessary to know the relationship between the compressible and incompressible velocities at each point. This is discussed in

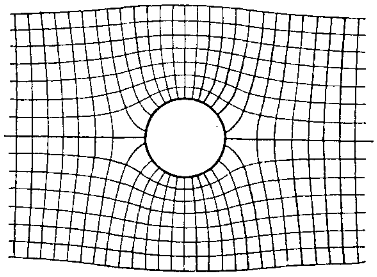

Fig. 1 Compressible flow.

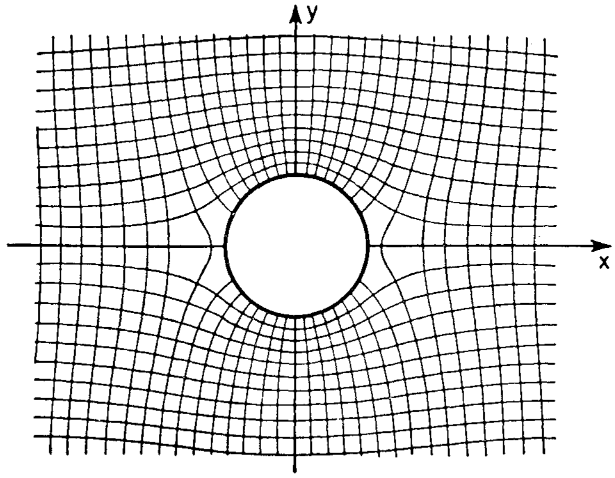

Fig. 2 Incompressible flow.

more detail later. Knowing this velocity relationship allows the calculation of the ϕ_i vs s_i distribution for the incompressible case. Conformal transformation techniques then may be used to determine the equivalent incompressible flow as follows.

Consider the transformation of the airfoil into a circle by the equation $Z_i = f(\zeta)$ where Z_i represents the incompressible airfoil plane, and ζ represents the circle plane. It can be shown that

$$\log(dZ_i/d\zeta) = \log|ds_i/d\theta| + i(\alpha - \theta - \pi/2)$$

which is an analytic function of ζ.

The form of the mapping function derivative is chosen to be

$$dZ_i/d\zeta = (1 - \zeta_T/\zeta)^{(1-\epsilon_T/\pi)}(a_0 + a_1/\zeta + a_2/\zeta 2 + a_3/\zeta 3 + \ldots)$$

The quantity $(1 - \zeta_T/\zeta)^{(1-\epsilon_T/\pi)}$ is introduced to allow a singularity existing at the trailing edge ζ_T to be treated separately, leaving an analytic expression without singularities to be evaluated.

The expression

$$\log|ds_i/d\theta| - \log|(1 - \zeta_T/\zeta)^{(1-\epsilon_T/\pi)}|$$

is expanded in a Fourier series with θ.

The complex conjugate of this series represents

$$(\alpha - \theta - \pi/2) - \arg(1 - \zeta_T/\zeta)^{(1-\epsilon_T/\pi)}$$

allowing α to be determined.

Using these values of α, the compressible airfoil shape is determined. Closure of the airfoil is accomplished by suitable choice of trailing-edge angle and leading-edge stagnation point.

The problem of determining a velocity transformation function V_c/V_i will now be discussed.

The equations of motion for a compressible fluid in the hodograph plane are[6]

$$\partial\phi_c/\partial\alpha = f(\partial\psi_c/\partial V_c),\ g(\partial\phi_c/\partial V_c) = -\partial\psi_c/\partial\alpha \quad (1)$$

where

$$f = (\rho_0/\rho)V_c,\ g = -1/V_c[(d/dV_c)(\rho_0/\rho V_c)]$$

and the incompressible hodograph equations:

$$\partial\varphi/\partial\alpha = V_i\partial\psi/\partial V_i \quad (2a)$$

$$(\partial\varphi/\partial V_i)V_i = -\partial\psi/\partial\alpha \quad (2b)$$

In order to transform Eqs. (1–2) by a substitution V_i for V_c set

$$dV_i/V_i = dV_c/f = dV_c/g \quad (3)$$

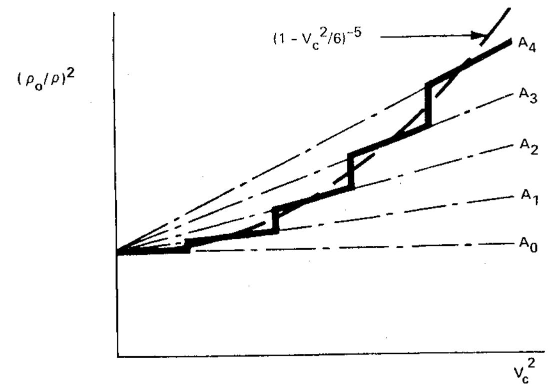

Fig. 3 Density-velocity relationships.

and f must equal g, and the following steps can be performed:

$$
\begin{aligned}
(\rho_0/\rho)V_c^2[(d/dV_c)(\rho_0/\rho V_0)] &= -1 \\
(\rho_0/\rho V_c)[d(\rho_0/\rho V_c)] &= -dV_c/V_c^3 \\
(\tfrac{1}{2})[d(\rho_0/\rho V_c)^2] &= -dV_c/V_c^3 \qquad (4) \\
(\tfrac{1}{2})(\rho_0/\rho V_c)^2 &= \tfrac{1}{2}V_c^2 + A' \\
(\rho_0/\rho)^2 &= 1 + AV_c^2
\end{aligned}
$$

Using Eq. (3)

$$dV_i/V_i = dV_c/V_c(1 + AV_c^2)^{1/2}$$

Integrating gives

$$V_i/V_c = K/[1 + (1 + AV_c^2)^{1/2}] \qquad (5)$$

Since the isentropic flow of air ($\gamma = 1.4$) does not obey Eq. (4), let A take on various values over small velocity ranges, such that Eq. (4) will give the proper value of ρ_0/ρ for air. This requires

$$1 + AV_c^2 = (1 - V_c^2/6)^{-5} = (\rho_0/\rho)^2 \qquad (6)$$

The exact variation of ρ_0/ρ vs V_c for air will then be approximated as shown in Fig. 3. K is set equal to 2 to make $V_i/V_c = 1$ as M approaches zero and therefore,

$$V_i/V_c = 2/[1 + (\rho_0/\rho)] \qquad (7)$$

The correct values of (ρ_0/ρ) for the isentroxic flow of air are used in this velocity transformation equation.

The physical interpretation of this method is that the real gas is approximated with many small regions of a fictitious gas which could be called a "traife gas." The fictitious gas does not behave like the real gas over an extended area, but in each small region it has the same density as air.

It is important to distinguish between the present traife gas and the well-known "tangent gas" or "$\gamma = -1$ gas." The present gas has no pressure, temperature, speed of sound, or Mach number. Traife gas is only a mathematical tool to duplicate the desired density vs velocity relationship. Any attempt to use its other physical characteristics seems to cause trouble.

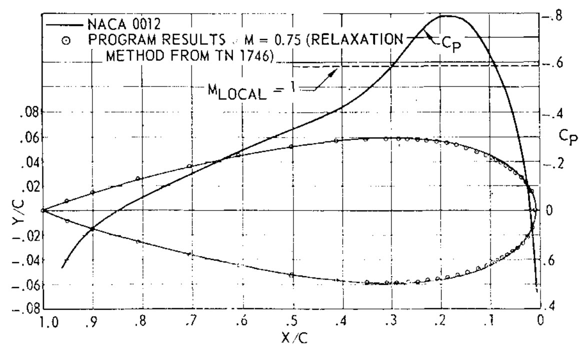

Fig. 4 Airfoil calculation from supercritical relaxation data.

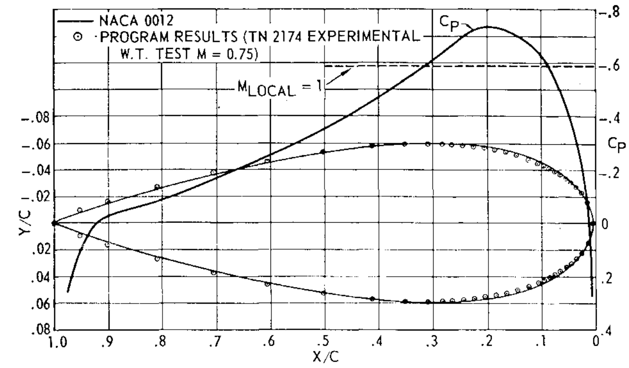

Fig. 5 Airfoil calculation from supercritical experimental data.

It is true that the rate of change of density with velocity is important to the hodograph equations. However, this variation is not contained in the basic physics of the problem. It comes about by mathematical manipulation in deriving the hodograph equations from the physical equations.

It is therefore not surprising that by further mathematical manipulation the variation of density with velocity can be ignored and only the absolute value of the density may be important.

To look at this issue from a different point of view, consider a two-dimensional electric analogy tank with a wax bottom. Such an apparatus was constructed by Taylor.[7] If the wax can be carved to varying depths to simulate the proper density, the compressible problem can be solved. It probably won't matter too much if the edge of the knife which cuts the wax is sawtoothed, especially if the teeth can be made extremely fine. This physical experiment implies that only the absolute value of the density is important.

Comparisons with Existing Data

The method previously described for supercritical airfoil design has been programed for IBM 360/65 system in basic FORTRAN IV. The program is relatively simple to use and requires only a few seconds machine time to run one case.

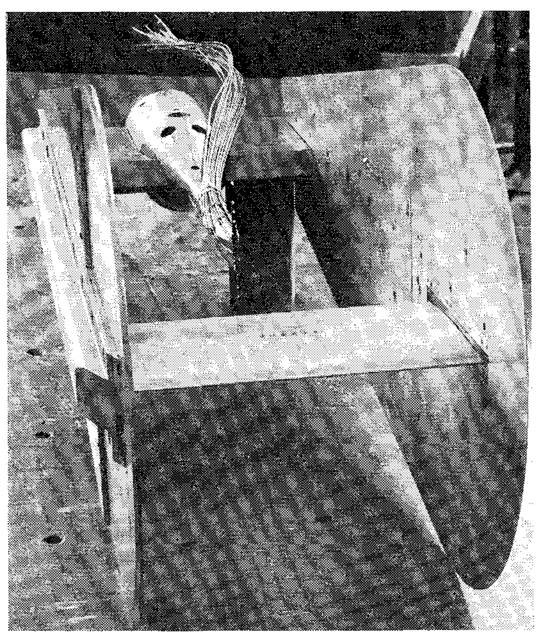

Fig. 6 Airfoil model and support system.

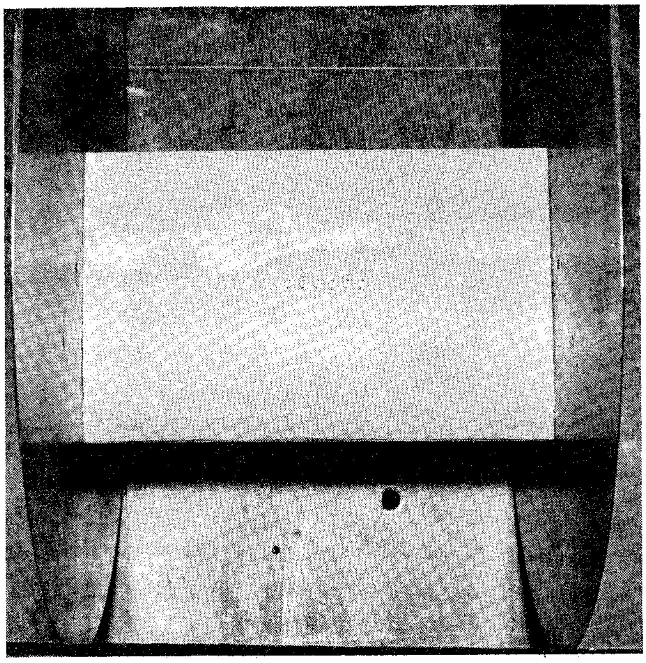

Fig. 7 Airfoil model, top view.

The accuracy of the program for supercritical flows is shown in Fig. 4, where the results of calculations for an NACA 0012 section at 0.75 Mach number are given. The input data are calculated pressure distributions using a relaxation technique performed by Emmonds and presented in NACA TN 1746. The results indicate that the method is feasible and can give good results. A comparison with supercritical experimental data for a NACA 0012 airfoil, Fig. 5, shows the computed shape to grow thicker toward the trailing edge. This is believed to be due to the boundary-layer displacement thickness on the actual experimental airfoil. It should be noted that supercritical flow exists on the NACA 0012 airfoil at the Mach number tested. The local Mach number is above 1 from 10–30% chord and reaches a peak of approximately 1.1.

The program has been applied to other supercritical transonic airfoils with more extensive supersonic velocities, and the results indicate similar accuracy to that obtained for the NACA 0012. Among these airfoils were asymmetrical and lifting configurations.

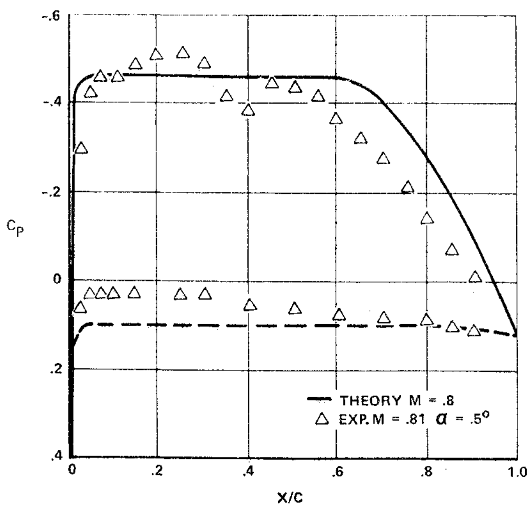

Fig. 8 Comparison of theoretical and experimental pressure distribution.

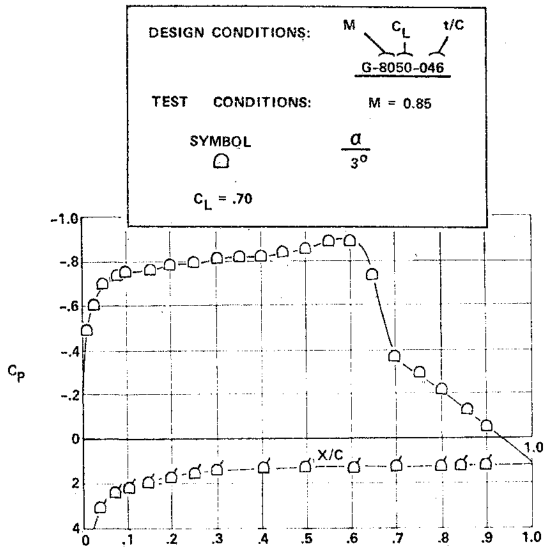

Fig. 9 Experimental pressure distribution.

Application to Airfoil Design

Using the supercritical airfoil design program, a two-dimensional airfoil was designed and constructed for wind-tunnel testing. The airfoil was designated G-8050-046.

The airfoil was installed between end plates in a 2 × 2 ft transonic blowdown wind tunnel. The experimental setup is shown in Figs. 6 and 7. Surface pressures and wake pressures were measured at angles of attack from minus 2° to plus 5° through a Mach number range of 0.48–1.1.

The airfoil was designed to have a C_L of approximately 0.5 at a Mach number of 0.8. Figure 8 shows a comparison of the design pressure distribution with experimental results. The agreement is considered excellent. Most of the discrepancies are believed to be a result of model inaccuracies which were unavoidable because of the small scale of the tests and the boundary-layer effects.

Because the model was conservatively designed to perform at $M = 0.80$, good results were obtained at higher speeds. Figure 9 shows that trailing-edge pressure divergence (an indication of buffet onset) was avoided even up to $C_L = 0.7$ at $M = 0.85$. Wake drag polars are shown in Fig. 10 for various Mach numbers.

To shed more light on buffet onset at transonic speeds, Fig. 11 is presented. The pressure at the 90% station is

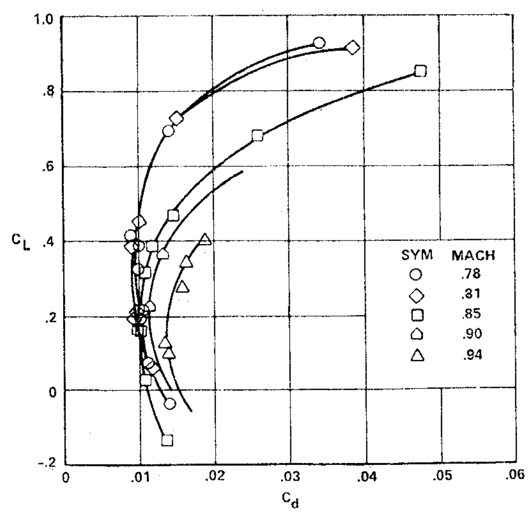

Fig. 10 Wake drag polars.

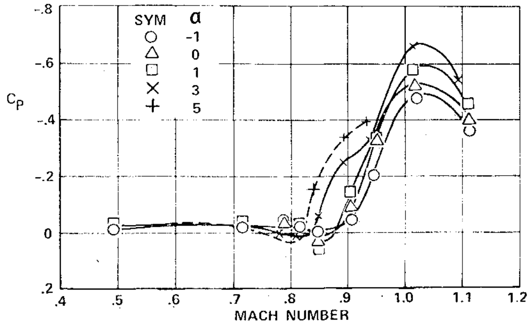

Fig. 11 Trailing-edge pressure divergence.

plotted against Mach number. Buffet onset is assumed to be indicated by the sharp break in these pressures.

Pressure divergence points from Fig. 11 are shown in Fig. 12 in the form of C_L vs Mach number. This plot indicates that at the higher Mach numbers the newly designed airfoil can reach lift coefficients higher than typical present-day designs.

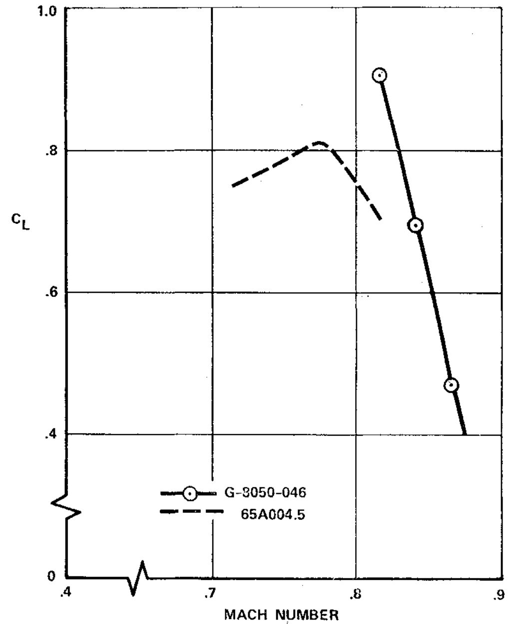

Fig. 12 Trailing-edge pressure divergence vs lift coefficient.

Conclusions

Results of calculations of airfoil shapes from supercritical compressible velocity distributions indicate that it is feasible to calculate the shapes with reasonable accuracy using a transformation to an incompressible plane, and that the method can be useful for studying transonic airfoil shapes with supercritical velocities.

Some velocity distributions may not be physically realized, and the shapes computed by this method may not always produce the assumed velocity distribution. They could produce a flow with a shock wave that is not in the assumed velocity distribution. Other criteria, such as a limit line analysis,[8] boundary-layer calculations, and airfoil design experience are necessary.

The results of the design of airfoil shape G-8050-046, developed by the Transonic Airfoil Design Program, and its subsequent testing, indicate that the theoretical method of developing the geometry of an airfoil from the velocity or pressure distribution agrees well with experimental data.

It is concluded that good airfoil designs can be obtained by using this existing program to calculate airfoil ordinates for a given velocity distribution. Although no special effort was made to optimize the performance of the section, it had better performance than comparable state-of-the-art airfoils, and there is room for further optimization. Thus, it seems that the airfoil design method can be incorporated in future aircraft design programs and future research and testing of transonic sections.

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